DELEGATE BOOKLET 1

(TASKS)

Pearson Edexcel International GCSE Further Pure Mathematics:

Effective Delivery and Assessment

4PM1

About this event

Course Title: IGCSE further Pure Mathematics Assessment and Delivery

Course Code:

Aims and Objectives of the event

To give an overview of the course
Consider some key elements of 4PM1
Review some examples of recent work and apply the mark scheme
Address common issues and FAQs

Agenda

Time	Item
9:30 – 10:00	Welcome Tea & Coffee
10:00	Agenda & Introductions
10:15 – 10:45	Logarithms and indices
10:45 – 11:15	Quadratic function and Identities and inequalities
11:15 – 12:30	Task 1, Graphs, series and Vectors
12:30 – 1:15	Lunch
1:15 – 2:30	Rectangular Cartesian coordinates, Calculus and Task 2
2:30 – 3:30	Trigonometry and Task 3
3:30 – 3:45	Open discussion and close

Activity 1 – Marking exercise - Q9

Purpose:

Review some examples of recent work and apply the mark scheme

Task 1

- Work through Question 9 Paper 1 from June 2016 (on next page)
- Use the mark scheme to mark the three student attempts at question 9

You will need the mark scheme booklet.

9 $$f(x) = 3x^2 - 5x - 4$$

The roots of the equation $f(x) = 0$ are α and β

(a) Without solving the equation $f(x) = 0$, form an equation, with integer coefficients, which has

(i) roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$

(6)

(ii) roots $2\alpha + \beta$ and $\alpha + 2\beta$

(5)

(b) Express $f(x)$ in the form $A(x + B)^2 + C$, stating the values of the constants A, B and C.

(3)

(c) Hence, or otherwise, show that the equation $f(x) = -8$ has no real roots.

(2)

Student example 1

9 $$f(x) = 3x^2 - 5x - 4$$

The roots of the equation $f(x) = 0$ are α and β

(a) Without solving the equation $f(x) = 0$, form an equation, with integer coefficients, which has

(i) roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ **(6)**

(ii) roots $2\alpha + \beta$ and $\alpha + 2\beta$ **(5)**

(b) Express $f(x)$ in the form $A(x + B)^2 + C$, stating the values of the constants A, B and C. **(3)**

(c) Hence, or otherwise, show that the equation $f(x) = -8$ has no real roots. **(2)**

a) i) $\alpha\beta = -\frac{4}{3}$

$\alpha + \beta = \frac{5}{3}$

$\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = \frac{c}{a} = 1$ $\leftarrow \frac{a^2}{\alpha\beta} + \frac{b^2}{\alpha\beta}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = -\frac{b}{a}$ $(\alpha+\beta)^2 - 2\alpha\beta = \alpha^2+\beta^2$

$\frac{\left(\frac{5}{3}\right)^2 + \frac{8}{3}}{\frac{4}{3}} = \frac{49}{12}$

$\frac{49}{12} \times 12 = 49$

$12x^2 - 49x + 12 = 0$

ii) $(2\alpha+\beta)(\alpha+2\beta) = \frac{c}{a}$

$2\alpha^2 + 2\beta^2 + 5\alpha\beta = 2(\alpha^2+\beta^2) + 5\alpha\beta = \frac{c}{a}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{5}{3}\right)^2 + \frac{8}{3} = \frac{49}{9}$

$2\alpha+\beta+\alpha+2\beta = 3\alpha+3\beta = -\frac{b}{a}$

$\frac{5}{3} \times 3 = 5 = -\frac{b}{a}$

$9 = a,\ c = 49,\ b = -9\times5 = -45$

$9x^2 - 45x + 49 = 0$

P 4 6 9 0 1 A 0 2 6 3 2

Question 9 continued

b) $f(x)=3x^2-5x-4$

$A(x+B)^2+C=Ax^2+2ABx+AB^2+C$

$Ax^2+2ABx+AB^2+C=3x^2-5x-4$

$-4=AB^2+C$

$-5=2AB$

$3=A$

$-5=6B$

$B=-\frac{5}{6}$

$\left(-\frac{5}{6}\right)^2=\frac{25}{36}+C=-4$

$C=-4\frac{25}{36}$

$3\left(x-\frac{5}{6}\right)^2-\frac{73}{12}$

c) $3x^2-5x-4=-8$

$3x^2-5x+4=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\sqrt{25-48}$ is a root of a negative therefore $f(x)=-8$ has no real roots

Student example 2

9 $$f(x) = 3x^2 - 5x - 4$$

The roots of the equation $f(x) = 0$ are α and β

(a) Without solving the equation $f(x) = 0$, form an equation, with integer coefficients, which has

(i) roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ **(6)**

(ii) roots $2\alpha + \beta$ and $\alpha + 2\beta$ **(5)**

(b) Express $f(x)$ in the form $A(x + B)^2 + C$, stating the values of the constants A, B and C. **(3)**

(c) Hence, or otherwise, show that the equation $f(x) = -8$ has no real roots. **(2)**

a)i) $f(x) = 3x^2 - 5x - 4$

$\alpha\beta = -\frac{4}{3}$

$\alpha + \beta = \frac{5}{3}$

$x^2 + \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha\beta}{\alpha\beta}\right)$

$x^2 + \left(\frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta}\right)x + 1$

$-\frac{4}{3}$

$x^2 + \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)x + 1$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2$

$(\alpha+\beta)(\alpha+\beta)$

$\alpha^2 + \beta^2 + 2\alpha\beta$

$\left(-\frac{4}{3}\right)^2 - 2 \times -\frac{4}{3}$

$\frac{16}{9} + \frac{8}{3} = \frac{40}{9}$ $\qquad \frac{40}{9} \div \frac{5}{3} = \frac{8}{3}$

$x^2 + \frac{8}{3} + 1$

$\cancel{3x^2 + 8x + 3}$ $\qquad 3x^2 + 8x + 3$

P 4 6 9 0 1 A 0 2 6 3 2

Question 9 continued

c) $f(x) = -8$ No real roots if $b^2-4ac<0$

$-8 = 3x^2 - 5x - 4$

$0 = 3x^2 - 5x + 4$

$(-5)^2 - 4\times3\times4 = 25-48 = -23$

-23 issmaller than 0 so there are no real roots.

b) $A(x+B)^2 + C$

$= A(x+B)(x+B) + C$

$= A(x^2 + 2Bx + B^2) + C$

$= Ax^2 + 2ABx + AB^2 + C$ ①

$A = 3$ sub $A=3$ into ①

$3x^2 + 2\times3\times Bx + 3\times B^2 + C$ ②

$B = -\frac{5}{6}$ sub $B = -\frac{5}{6}$ into ②

$3x^2 \cancel{+} -5x + \frac{25}{12} + C$

$C = \frac{23}{12}$

a) ii) $x^2 + (2\alpha+\beta+\alpha+2\beta)x + (2\alpha+\beta)(2\beta+\alpha)$

$x^2 + (3\alpha+3\beta)x + 2\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta)$

$x^2 + 3(\frac{5}{3})x + (-\frac{4}{3}\times2 + \frac{40}{9}\times2 + -\frac{4}{3})$

$x^2 + 5x + (-\frac{8}{3} + \frac{80}{9} + -\frac{4}{3})$

$x^2 + 5x + \frac{44}{9}$

$9x^2 + 45x + 44$

Student example 3

9 $$f(x) = 3x^2 - 5x - 4$$

The roots of the equation $f(x) = 0$ are α and β

(a) Without solving the equation $f(x) = 0$, form an equation, with integer coefficients, which has

(i) roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ **(6)**

(ii) roots $2\alpha + \beta$ and $\alpha + 2\beta$ **(5)**

(b) Express $f(x)$ in the form $A(x + B)^2 + C$, stating the values of the constants A, B and C. **(3)**

$b^2 - 4ac$

(c) Hence, or otherwise, show that the equation $f(x) = -8$ has no real roots. **(2)**

$\alpha + \beta = -\frac{b}{a} = \frac{5}{3} \qquad \alpha\beta = \frac{c}{a} = -\frac{4}{3}$

i) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \dfrac{\left(\frac{5}{3}\right)^2 - 2\times\left(-\frac{4}{3}\right)}{\left(-\frac{4}{3}\right)}$

$\dfrac{\frac{25}{9} + \frac{8}{3}}{-\frac{4}{3}} = -\frac{49}{12} = -\frac{b}{a} \qquad \frac{b}{a} = \frac{49}{12}$

$\dfrac{\alpha}{\beta} \times \dfrac{\beta}{\alpha} = \dfrac{\alpha\beta}{\alpha\beta} = \dfrac{1}{1} \quad \therefore \dfrac{c}{a} = 1$

$x^2 + \frac{49}{12}x + \frac{1}{1} = 0$

$12x^2 + 49x + 12 = 0$

P 4 6 9 0 1 A 0 2 6 3 2

Question 9 continued

ii) $(2\alpha+\beta)+(\alpha+2\beta) = -\frac{b}{a}$

$3\alpha+3\beta = -\frac{b}{a} = 3(\alpha+\beta) = 3\times\frac{5}{3} = 5$

$\therefore -\frac{b}{a} = 5 \qquad -5 = \frac{b}{a}$

$(2\alpha+\beta)(\alpha+2\beta) = \frac{c}{a} = 2\alpha^2+\alpha\beta+4\alpha\beta+2\beta^2$

$\frac{c}{a} = 2\alpha^2+2\beta^2+5\alpha\beta$

$\frac{c}{a} = 2(\alpha^2+\beta^2) + 5(\alpha\beta)$

$= 2((\alpha+\beta)^2-2\alpha\beta) + 5(-\frac{4}{3})$

$= 2((\frac{5}{3})^2 - 2\times(-\frac{4}{3})) + \frac{20}{3} = (2\times\frac{49}{9}) + \frac{20}{3} = \frac{c}{a}$

$\frac{98}{9} + \frac{20}{3} = \frac{158}{9} = \frac{c}{a}$

$x^2 \cancel{+} - 5x + \frac{158}{9} = 0$

$9x^2 - 45x + 158 = 0$

b) $f(x) = \cancel{3x}\ \cancel{3(x+1)^2}$

$\cancel{(x+1)^2 = x^2+2x+1}$

$\cancel{3x^2+6x+3}$

$\cancel{(x-1)^2 = x^2-2x+1} \qquad \cancel{3(x+\frac{5}{3})^2}$

$\cancel{3x^2-6x+3} \qquad \cancel{3(x^2}$

$\cancel{+x+} \qquad \frac{10}{3}$

Question 9 continued

c) if $b^2 - 4ac < 0$ there are no real roots

$\cancel{(-5)^2 - (4 \times 3 \times (-4)}$)

$3x^2 - 5x - 4 = -8$

$3x^2 - 5x + 4 = 0$

$(-5)^2 - (4 \times 3 \times 4)$

$= 25 - 48 = -23$

$-23 < 0 \therefore$ no real roots

Activity 2 – Marking exercise - Q4

Purpose:

Review some examples of recent work and apply the mark scheme

Task 1

- Work through Question 4 Paper 2 from June 2016 (on next page)
- Use the mark scheme to mark the three student attempts at question 4

You will need the mark scheme booklet.

4 Given that $y = e^{2x}\sqrt{x+1}$

show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(4x+5)}{2\sqrt{x+1}}$

(6)

Student example 1

4 Given that $y = e^{2x}\sqrt{x+1}$

show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(4x+5)}{2\sqrt{x+1}}$ **(6)**

$y = e^{2x}\sqrt{x+1}$

$y = e^{2x}\,(x+1)^{-\frac{1}{2}}$

$y = \dfrac{e^{2x}}{(x+1)^{\frac{1}{2}}}$

$\dfrac{dy}{dx} = \dfrac{(x+1)^{\frac{1}{2}}\,2e^{2x} - e^{2x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(x+1)^{\frac{1}{2}(2)}}$

$= \cancel{\sqrt{x+1}\,(2e^{2x}\ e^{2x}}$

$= \dfrac{\sqrt{x+1}\,2e^{2x} - e^{2x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{x+1}$

$= \dfrac{\left[2(x+1)^{\frac{1}{2}} - \left(\frac{1}{2}x^{-\frac{1}{2}}\right)\right]e^{2x}}{x+1}$

$= \dfrac{\left(2\sqrt{x+1} - \frac{1}{2}x^{-\frac{1}{2}}\right)e^{2x}}{x+1}$

$=$

$\cancel{y = e^{2x}(x+1)^{\frac{1}{2}}}$

$\dfrac{dy}{dx}\ \cancel{= e^{2x}\,\frac{1}{2}(1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}\,2e^{2x}}$

$= e^{2x}\left(\frac{1}{2}\right)^{-\frac{1}{2}} + \left(\sqrt{x+1}\right)2e^{2x}$

$= \left(\left(\frac{1}{2}\right)^{-\frac{1}{2}} + 2\sqrt{x+1}\right)e^{2x}$

$= \dfrac{\sqrt{2}}{2} + 2\sqrt{x+1}$

$= \dfrac{\sqrt{2} + 4\sqrt{x+1}}{2}$

6

P 4 6 2 4 1 A 0 6 3 6

Question 4 continued

$y = e^{2x}(x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{e^{2x}}{(x+1)^{-\frac{1}{2}}}$

$= \frac{(x+1)^{-\frac{1}{2}} 2e^{2x} - e^{2x}(-\frac{1}{2})(1)}{((x+1)^{-\frac{1}{2}})^2}$

$= \frac{(x+1)^{-\frac{1}{2}}(2e^{2x}) - e^{2x}(-\frac{1}{2})}{(x+1)^{-1}}$

$= \frac{(2(x+1)^{-\frac{1}{2}} + \frac{1}{2})e^{2x}}{(x+1)^{-1}}$

$= \frac{(x+1)e^{2x}}{(2(x+1)^{-\frac{1}{2}} + \frac{1}{2})^{-1}}$

$= \frac{(x+1)e^{2x}}{\frac{1}{2}\sqrt{x+1} + 2}$

$= \frac{(4x+4)e^{2x}}{2\sqrt{x+1} + 2}$

(Total for Question 4 is 6 marks)

Student example 2

4 Given that $y = e^{2x}\sqrt{x+1}$

show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(4x+5)}{2\sqrt{x+1}}$

(6)

a) $y = e^{2x}(x+1)^{\frac{1}{2}}$

$du = 2e^{2x}$

$dv = \frac{1}{2}(x+1)$

$\frac{1}{2}(x+1)^{-\frac{1}{2}}.$

$= e^{2x}\left(\frac{1}{2}x+1\right)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}(2e^{2x})$

$= \cancel{\frac{1}{2}}\, e^{2x}\, \cancel{\left(\frac{1}{2}x+1\right.}$

$\cancel{\frac{1}{2}e^{2x}\left[\left(x+1\right)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}(4)\right]}$

$\cancel{\frac{1}{2}e^{2x}\ \frac{(x+1)(4)}{\sqrt{x+1}}}$

$\cancel{\frac{1}{2}e^{2x}\ x^2 +}$

$e^{2x}\left(\frac{1}{2}x+1\right)^{-\frac{1}{2}} + 2e^{2x}(x+1)^{\frac{1}{2}}$

$\cancel{\frac{1}{2}e^{2x}\Big[}$

$e^{2x}\left[\left(\frac{1}{2}x+1\right)^{-\frac{1}{2}} + 2(x+1)^{\frac{1}{2}}\right.$

$e^{2x}\left[\left(\frac{1}{2}x+1\right)^{-\frac{1}{2}} + 4(x+1)^{\frac{1}{2}}\right.$

$\dfrac{e^{2x}(}{2}$

P 4 6 2 4 1 A 0 6 3 6

DO NOT WRITE IN THIS AREA

Question 4 continued

Q4 a) $y = e^{2x}(x+1)^{\frac{1}{2}}$

$du = 2e^{2x}$

$dv = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$\frac{dy}{dx}$ $e^{2x}\left(\frac{1}{2}(x+1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}(2e^{2x})\right)$

$e^{2x}\left\{\frac{1}{2}(x+1)^{-\frac{1}{2}} + 2e^{2x}(x+1)^{\frac{1}{2}}\right\}$

$\frac{1}{2}e^{2x}\left[(x+1)^{-\frac{1}{2}} + 4x(x+1)^{\frac{1}{2}}\right]$

$\cancel{\frac{1}{2}e^{2x}[\; 4x +}$

$\cancel{x - \frac{1}{2} - \frac{\frac{1}{2}(-\frac{1}{2}-1)(1)^2}{2!}}$

$\cancel{x - \frac{1}{2}}$

$\cancel{(x+1)^{\frac{1}{2}}}$

$\cancel{x + \frac{1}{2} + \frac{\frac{1}{2}(\frac{1}{2}-1)(1)^2}{2!}}$

$\cancel{x + \frac{1}{2} - \frac{1}{8}}$

$\cancel{x + \frac{3}{8}}$

$\cancel{\frac{1}{2}e^{2x}\left[\frac{4(x+\frac{3}{8})}{\sqrt{x+1}}\right]}$

$\frac{1}{2}e^{2x}\left[\frac{4x+5}{\sqrt{x+1}}\right]$

$= \frac{e^{2x}(4x+5)}{2\sqrt{x+1}}$

(Total for Question 4 is 6 marks)

Student example 3

4 Given that $y = e^{2x}\sqrt{x+1}$

show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(4x+5)}{2\sqrt{x+1}}$ (6)

$\cancel{\frac{dy}{dx} = \frac{dy}{dx}\ 2e^{2x}}$

$y = e^{2x}\sqrt{x+1}$

$y = e^{2x}(x+1)^{\frac{1}{2}}$

$f(x)$ $g(x)$

$f'(x)$ $\frac{dy}{dx} = 2e^{2x}$ $\frac{dy}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$g'(x) = \dfrac{1}{2\sqrt{x+1}}$

$\cancel{2e^{2x} = 2e^{2x}}$

$\cancel{2e^{2x}\ \frac{1}{2\sqrt{x+1}}\ 2e^{2x}\ \frac{1}{2}\sqrt{x+1}\ (2e^{2x})\ \frac{1}{2\sqrt{x+1}}(2e^{2x}}$

$$= \frac{\frac{1}{2}\sqrt{x+1}\,(2e^{2x}) - \frac{1}{2\sqrt{x+1}}(e^{2x})}{\left(\frac{1}{2\sqrt{x+1}}\right)^2}$$

$$= \frac{2e^{2x}\sqrt{x+1}}{2} \times \frac{e^{2x}\ \cancel{4e^{2x}}}{2\sqrt{x+1}}$$

$$= \frac{4e^{2x}(\sqrt{x+1})^2 - 2e^{2x}}{(4\sqrt{x+1})^2} = \cancel{2e^{2x}+1}$$

P 4 6 2 4 1 A 0 6 3 6

Question 4 continued

$\frac{2}{2}e^{3x}$

(Total for Question 4 is 6 marks)

Activity 3 – Question design

Purpose:

To adapt questions in order to increase/decrease the level of demand.
(Taylor-make some questions for your students)

Task

With the two questions given, change them to alter the level of demand.
How would your changes affect the total mark?
How would your changes affect the total mark scheme?

Question 6 from Paper 1 of the Sample Assessment Material 4PM1

Could this be altered to allow the less able student to access the question?

How would you amend the mark scheme?

$$y = e^x\left(x^2 - 3x\right)$$

Show that $y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} = 2e^x$

(8)

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This is part of Question 8 Paper 1 June 2015

How would you add to the question to extend the more able? How would you amend the mark scheme?

Using the identities

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

(a) (i) show that $\cos 2A = 1 - 2\sin^2 A$

(3)

(ii) express $\sin 2A$ in terms of $\sin A$ and $\cos A$ simplifying your answer

(1)

(b) Hence show that $\sin 3A = 3\sin A - 4\sin^3 A$

(4)

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